

# DESIGN OF VAN HIELE'S LEVEL 5 INDICATORS USING THE DELPHI METHODOLOGY

Alberto Arnal-Bailera, Víctor Manero

Mathematics Department, University of Zaragoza

The van Hiele model (van Hiele, 1986) establishes five levels of development of geometric thought, from level 1 (visual) to level 5 (rigor). Despite the fact that the van Hiele model has been deeply studied, there are few research works concerning the fifth level. Our goal is to describe this level through the construction and validation of a list of indicators for each of the processes involved in geometrical reasoning. One of the few works about the fifth van Hiele level is the thesis of Stephen D. Blair where he states that working with level 5-activities promotes the development of lower levels "...explorations involving taxicab and spherical geometry support students' understanding within Euclidean geometry" (Blair, 2004, p. 334).

We are following the Delphi research methodology (Hsu & Sandford, 2007) that allows us to collect information from a panel of experts to reach a consensus through a series of phases. In the first phase, we collect information from this panel (Geometry researchers) analysing narratives in which they describe the reasoning at this level. With this information, we designed a list of tentative indicators of the level. For example, two experts described the proof abilities of a person who reasons at level 5 as "...can compare proofs of the same result" and "...analyses proofs to decide if, by modifying them, they can proof a different result". Based on the previous sentences, we have constructed the indicator "a person at level 5 can compare proofs on the basis of criteria such as the possibility of using it to prove more general statements".

We have obtained a list of 19 indicators concerning definition (6), proof (5), classification (4) and identification (4). In the following phases, an extended panel of experts will rate these indicators until a consensus is reached. The final product of the iterative application of this method is expected to be a list of indicators of the fifth van Hiele level of reasoning for each process.

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## References

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